Números Complejos

A complex number is represented in **rectangular form** as

$$z = x + jy \tag{2-1}$$

where, $j = \sqrt{-1}$ and (x, y) are real and imaginary coefficients of z respectively. We can treat (x, y) as a point in the **Cartesian** coordinate frame shown in Fig. 2-1. A point in a

Números Complejos

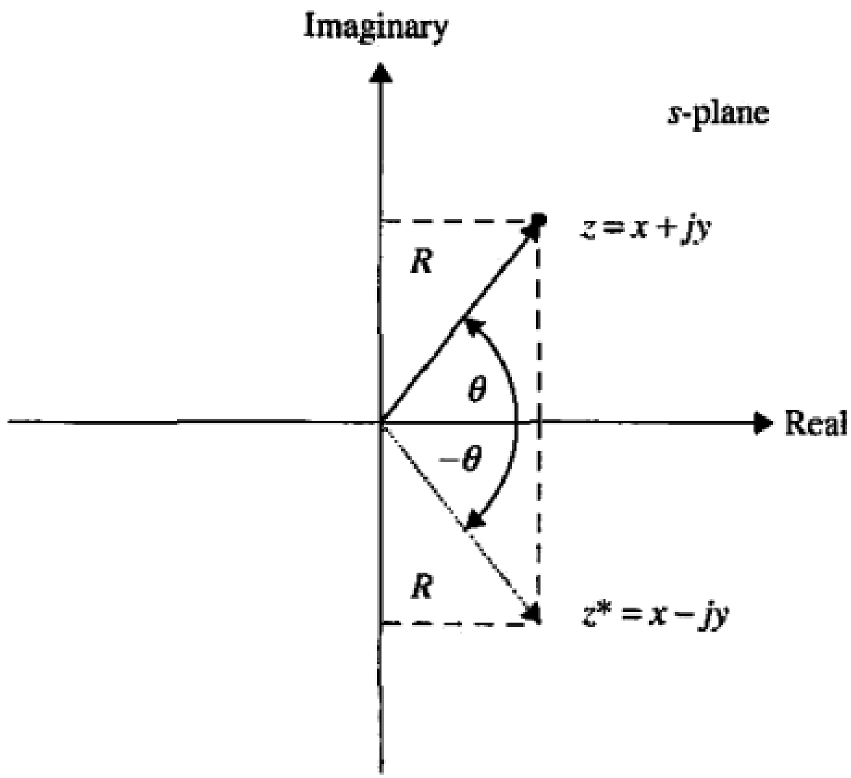

Figure 2-1 Complex number z representation in rectangular and polar forms.

rectangular coordinate frame may also be defined by a vector R and an angle θ. It is then easy to see that

$$\begin{aligned} x &= R\cos\theta \\ y &= R\sin\theta \end{aligned} \qquad (2\text{-}2)$$

Números Complejos

wnere,

$R =$ magnitude of z

$\theta =$ phase of z and is measured from the x axis. Right-hand rule convention: positive phase is in counter clockwise direction.

Hence,

$$R = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1} \frac{y}{x} \tag{2-3}$$

Introducing Eq. (2-2) into Eq. (2-1), we get

$$z = R\cos\theta + jR\sin\theta \tag{2-4}$$

Upon comparison of Taylor series of the terms involved, it is easy to confirm

$$e^{j\theta} = \cos\theta + j\sin\theta \tag{2-5}$$

Números Complejos

Eq. (2-5) is also known as the **Euler formula.** As a result, Eq. (2-1) may also be represented in **polar form** as

$$z = R\,e^{j\theta} = R\,\angle\,\theta \qquad (2\text{-}6)$$

We define the **conjugate** of the complex number z in Eq. (2-1) as

$$z^* = x - jy \qquad (2\text{-}7)$$

Or, alternatively,

$$z^* = R\cos\theta - jR\sin\theta = R\,e^{-j\theta} \qquad (2\text{-}8)$$

Note:

$$zz^* = R^2 = x^2 + y^2 \qquad (2\text{-}9)$$

Table 2-1 shows basic mathematical properties of complex numbers.

Funciones de Variable Compleja

The function $G(s)$ is said to be a function of the complex variable s if, for every value of s, there is one or more corresponding values of $G(s)$. Because s is defined to have real and imaginary parts, the function $G(s)$ is also represented by its real and imaginary parts; that is,

$$G(s) = \mathrm{Re}[G(s)] + j\,\mathrm{Im}[G(s)] \qquad (2\text{-}11)$$

Funciones de Variable Compleja

$$G(s) = \frac{1}{s(s+1)}$$

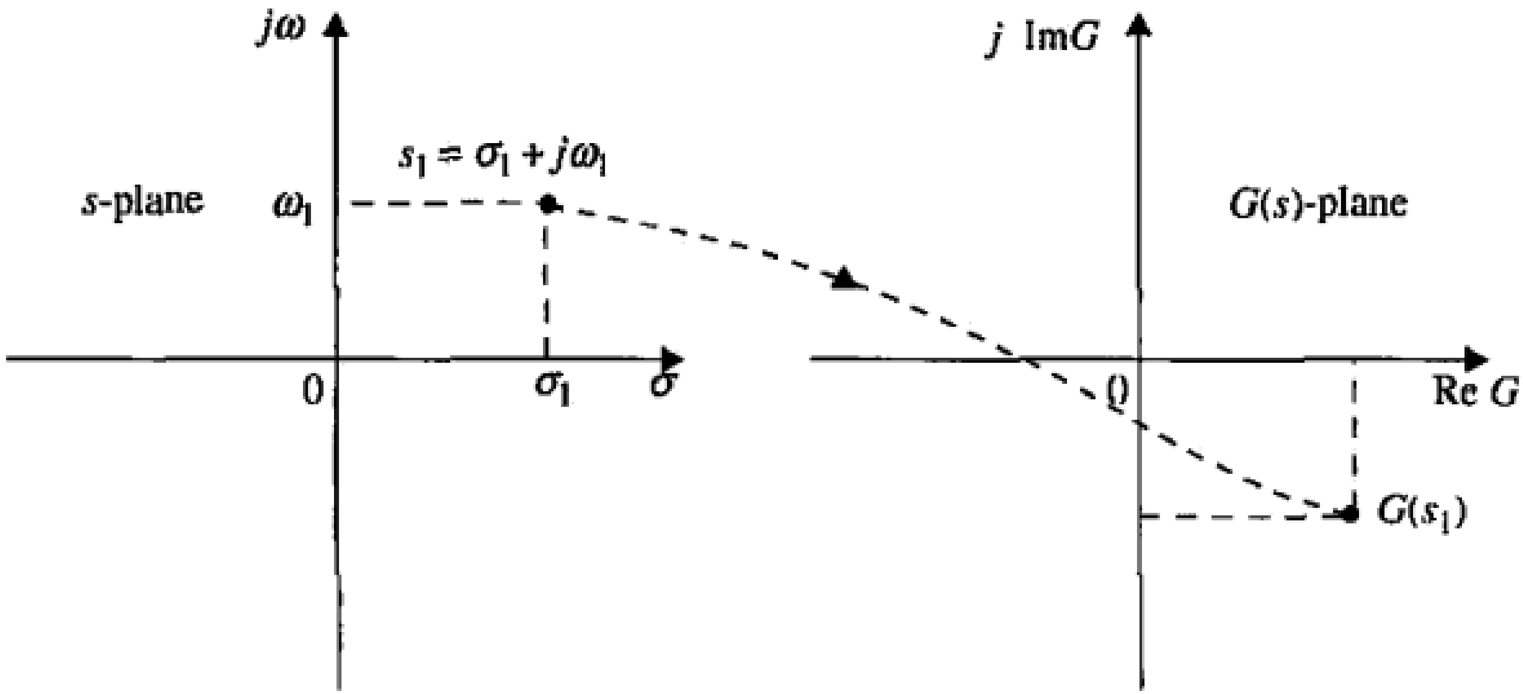

Figure 2-3 Single-valued mapping from the *s*-plane to the *G(s)*-plane.

Funciones de Variable Compleja

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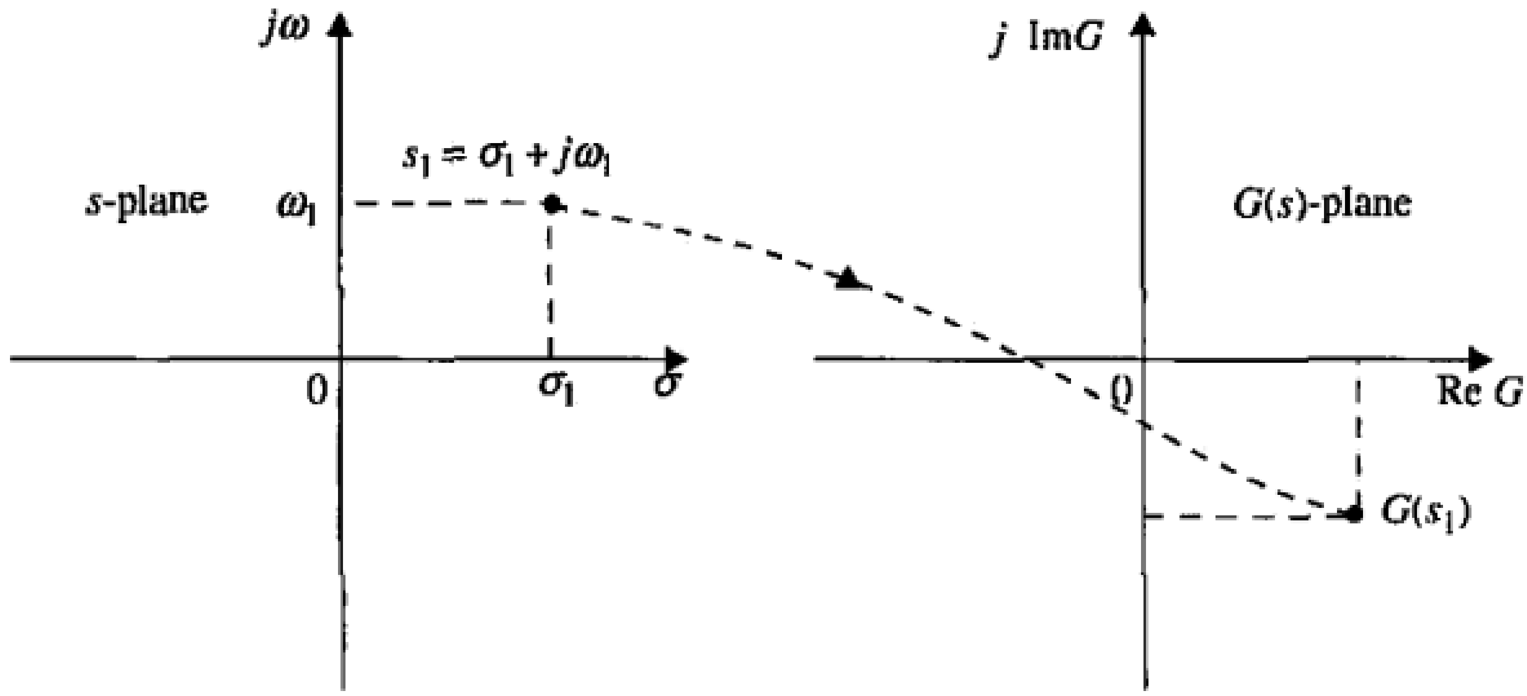

Figure 2-3 Single-valued mapping from the s-plane to the $G(s)$-plane.

Pierre-Simon Laplace

Pierre-Simon Laplace (Beaumont-en-Auge (Normandía);28 de marzo de 1749[1] - París; 5 de marzo de 1827) fue unastrónomo, físico y matemático francés que inventó y desarrolló la transformada de Laplace y la ecuación de Laplace. Fue un creyente del determinismo causal. (Wikipedia)

Transformada de Laplace

Given the real function $f(t)$ that satisfies the condition

$$\int_0^{\infty} \left| f(t)e^{-\sigma t} \right| dt < \infty \tag{2-115}$$

for some finite, real σ, the Laplace transform of $f(t)$ is defined as

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt \tag{2-116}$$

or

$$F(s) = \text{Laplace transform of } f(t) = \mathcal{L}[\,f(t)] \tag{2-117}$$

The variable s is referred to as the **Laplace operator**, which is a complex variable; that is, $s = \sigma + j\omega$, where σ is the real component and ω is the imaginary component. The defining equation in Eq. (2-117) is also known as the **one-sided Laplace transform**, as the integration is evaluated from $t = 0$ to ∞. This simply means that all information contained

Transformada de Laplace

EXAMPLE 2-4-1 Let $f(t)$ be a unit-step function that is defined as

$$f(t) = u_s(t) = 1 \quad t \geq 0$$
$$= 0 \quad t < 0 \tag{2-118}$$

The Laplace transform of $f(t)$ is obtained as

$$F(s) = \mathcal{L}[u_s(t)] = \int_0^\infty u_s(t)e^{-st}dt = -\frac{1}{s}e^{-st}\bigg|_0^\infty = \frac{1}{s} \tag{2-119}$$

Eq. (2-119) is valid if

$$\int_0^\infty \left|u_s(t)e^{-\sigma t}\right|dt = \int_0^\infty \left|e^{-\sigma t}\right|dt < \infty \tag{2-120}$$

which means that the real part of s, σ, must be greater than zero. In practice, we simply refer to the Laplace transform of the unit-step function as $1/s$, and rarely do we have to be concerned with the region in the s-plane in which the transform integral converges absolutely.

Transformada de Laplace

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Transformada de Laplace

■ **Theorem 1.** ***Multiplication by a Constant***
Let k be a constant and $F(s)$ be the Laplace transform of $f(t)$. Then

$$\mathcal{L}[kf(t)] = kF(s) \tag{2-125}$$

■ **Theorem 2.** ***Sum and Difference***
Let $F_1(s)$ and $F_2(s)$ be the Laplace transform of $f_1(t)$ and $f_2(t)$, respectively. Then

$$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s) \tag{2-126}$$

Transformada de Laplace

■ **Theorem 3.** ***Differentiation***

Let $F(s)$ be the Laplace transform of $f(t)$, and $f(0)$ is the limit of $f(t)$ as t approaches 0. The Laplace transform of the time derivative of $f(t)$ is

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - \lim_{t \to 0} f(t) = sF(s) - f(0) \qquad (2\text{-}127)$$

In general, for higher-order derivatives of $f(t)$,

$$\begin{aligned} \mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] &= s^n F(s) - \lim_{t \to 0}\left[s^{n-1} f(t) + s^{n-2}\frac{df(t)}{dt} + \cdots + \frac{d^{n-1} f(t)}{dt^{n-1}}\right] \\ &= s^n F(s) - s^{n-1} f(0) - s^{n-2} f^{(1)}(0) - \cdots - f^{(n-1)}(0) \end{aligned} \qquad (2\text{-}128)$$

where $f^{(i)}(0)$ denotes the ith-order derivative of $f(t)$ with respect to t, evaluated at $t = 0$.

Transformada de Laplace

Theorem 4. ***Integration***

The Laplace transform of the first integral of $f(t)$ with respect to t is the Laplace transform of $f(t)$ divided by s; that is,

$$\mathcal{L}\left[\int_0^t f(\tau)d\tau\right] = \frac{F(s)}{s} \tag{2-129}$$

For nth-order integration,

$$\mathcal{L}\left[\int_0^{t_n}\int_0^{t_{n-1}}\cdots\int_0^{t_1} f(t)d\tau dt_1 dt_2\cdots dt_{n-1}\right] = \frac{F(s)}{s^n} \tag{2-130}$$

Theorem 5. ***Shift in Time***

The Laplace transform of $f(t)$ delayed by time T is equal to the Laplace transform $f(t)$ multiplied by e^{-Ts}; that is,

$$\mathcal{L}[f(t-T)u_s(t-T)] = e^{-Ts}F(s) \tag{2-131}$$

where $u_s(t-T)$ denotes the unit-step function that is shifted in time to the right by T.

Transformada de Laplace

■ **Theorem 6.** ***Initial-Value Theorem***
If the Laplace transform of $f(t)$ is $F(s)$, then

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s) \tag{2-132}$$

if the limit exists.

■ **Theorem 7.** ***Final-Value Theorem***
If the Laplace transform of $f(t)$ is $F(s)$, and if $sF(s)$ is analytic (see Section 2-1-4 on the definition of an analytic function) on the imaginary axis and in the right half of the s-plane, then

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) \tag{2-133}$$

Transformada de Laplace

SOLUTION For a unit step input

$$f(t) = u_s(t) = \begin{cases} 0, & t < 0, \\ 1, & t \geq 0 \end{cases} \tag{2-180}$$

Eq. (2-179) is written as

$$u_s(t) = \tau\dot{y}(t) + y(t) \tag{2-181}$$

If $y(0) = \dot{y}(0) = 0$, $\mathcal{L}(u_s(t)) = \frac{1}{s}$ and $L(y(t)) = Y(s)$, we have

$$\frac{1}{s} = s\tau Y(s) + Y(s) \tag{2-182}$$

or

$$Y(s) = \frac{1}{s}\frac{1}{\tau s + 1} \tag{2-183}$$

Notice that the system has a pole at $s = -1/\tau$.

Using partial fractions, Eq. (2-183) becomes

$$Y(s) = \frac{K_0}{s} + \frac{K_{-1/\tau}}{\tau s + 1} \tag{2-184}$$

where, $K_0 = 1$ *and* $K_{-1/\tau} = -1$. Applying the inverse Laplace transform to Eq. (2-184), we get the time response of Eq. (2-179).

$$v_o(t) = 1 - e^{-t/\tau} \tag{2-185}$$

where t is the time for $y(t)$ to reach 63% of its final value of $\lim_{t \to \infty} y(t) = 1$.

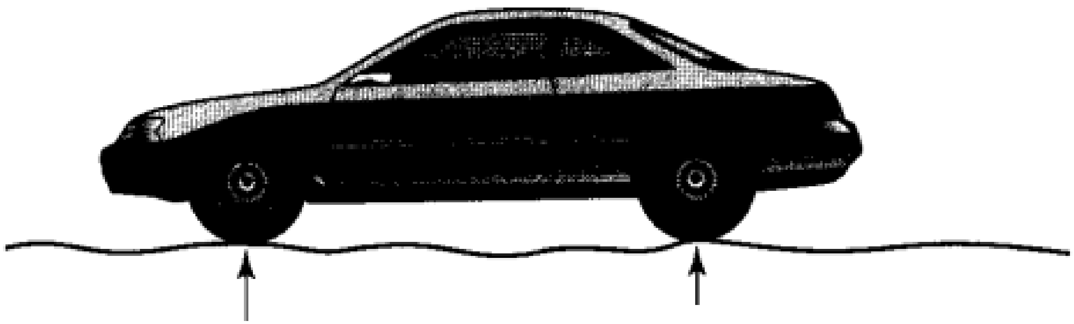
Función de Transferencia

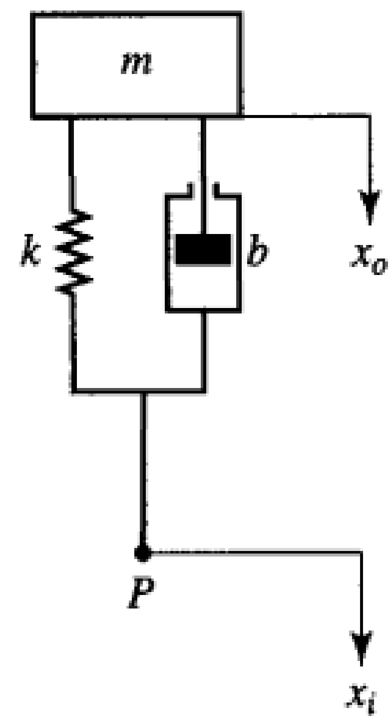
$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_m\frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + \cdots + b_1\frac{du(t)}{dt} + b_0 u(t) \tag{2-217}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

Condiciones Iniciales Nulas

Ejercicio





Ejercicio

Solución. La ecuación de movimiento para el sistema de la figura **3–35(b)** es

$$m\ddot{x}_o + b(\dot{x}_o - \dot{x}_i) + k(x_o - x_i) = 0$$

o bien

$$m\ddot{x}_o + b\dot{x}_o + kx_o = b\dot{x}_i + kx_i$$

Tomando la transformada de **Laplace** de esta última ecuación, y suponiendo condiciones iniciales de cero, obtenemos

$$(ms^2 + bs + k)X_o(s) = (bs + k)X_i(s)$$

Por tanto, la función de transferencia $X_o(s)/X_i(s)$ se obtiene mediante

$$\frac{X_o(s)}{X_i(s)} = \frac{bs + k}{ms^2 + bs + k}$$

Diagrama de Bloques

Diagramas de bloques. Un ***diagrama de bloques*** de un sistema es una representación gráfica de las funciones que lleva a cabo cada componente y el flujo de señales. Tal diagrama muestra las relaciones existentes entre los diversos componentes. A diferencia de una representación matemática puramente abstracta, un diagrama de bloques tiene la ventaja de indicar en forma más realista el flujo de las señales del sistema real.

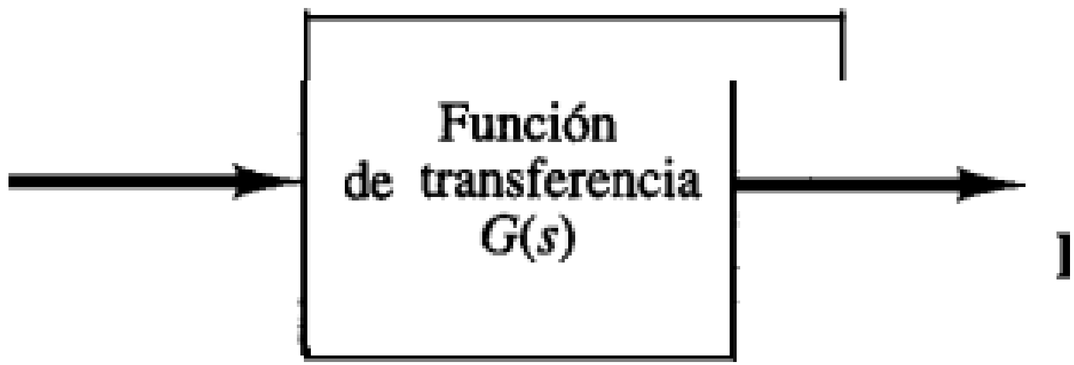


Diagrama de Bloques

do del punto de vista del análisis.

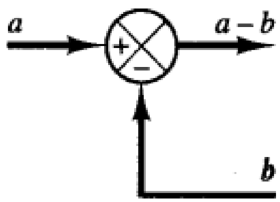


Figura 3-4
Punto suma.

Punto suma. Remitiéndonos a la figura 3-4, un círculo con una cruz es el símbolo que indica una operación de suma. El signo de más o de menos en cada punta de flecha indica si la señal debe sumarse o restarse. Es importante que las cantidades que se sumen o resten tengan las mismas dimensiones y las mismas unidades.

Punto de ramificación. Un punto *de ramificación* es aquel a partir del cual la señal de un bloque va de modo concurrente a otros bloques o puntos suma.

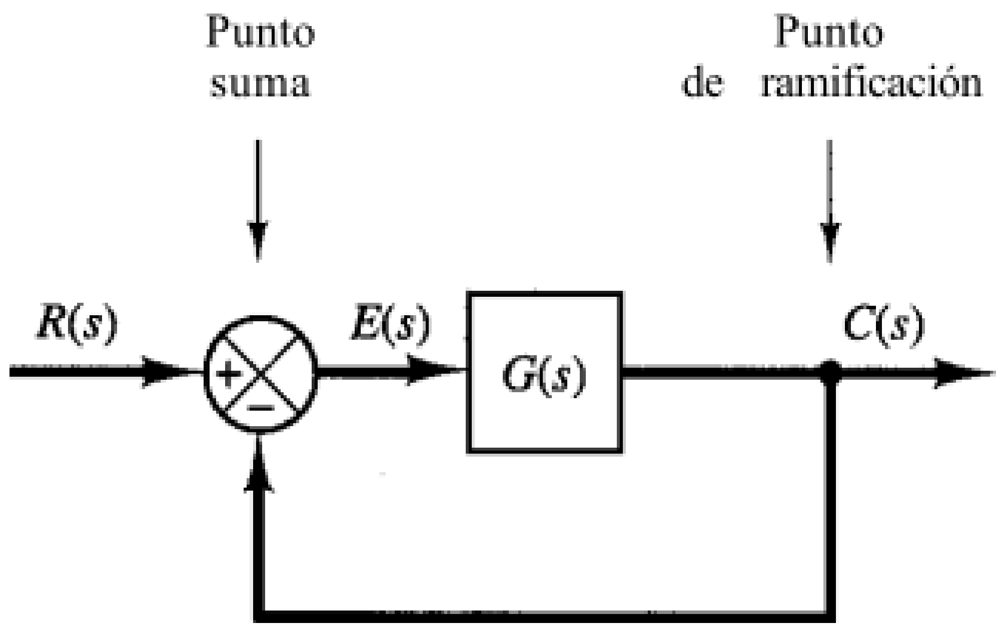


Diagrama de Bloques

Tabla 3-1 Reglas del álgebra de los diagramas de bloques

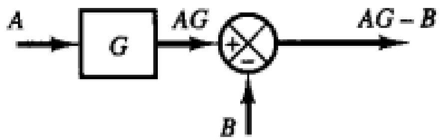
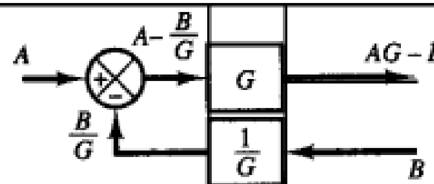
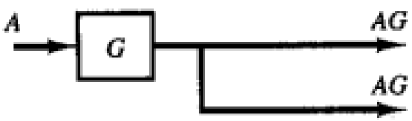
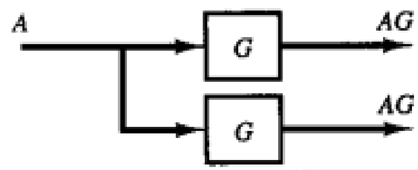
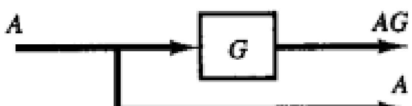
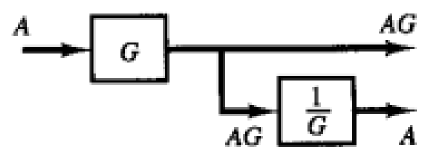
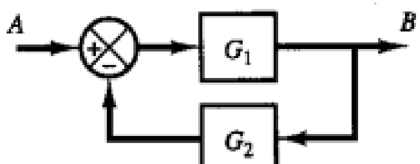
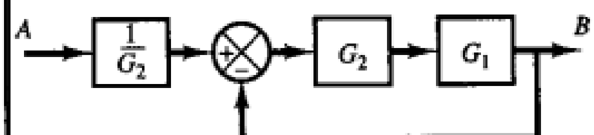
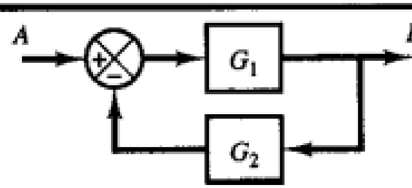
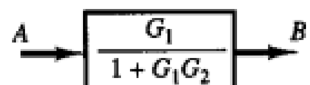
	Diaeramas de bloaues originales	Diagramas de bloques equivalentes
1		
2		
3		
4		
5		

Diagrama de Bloques

Considere el sistema que aparece en la figura 3–9(a). Simplifique este diagrama.

Si se mueve el punto suma del lazo de realimentación negativa que contiene H_2 hacia afuera del lazo de realimentación positiva que contiene H_1, obtenemos la figura 3–9(b). Si eliminamos

(a)

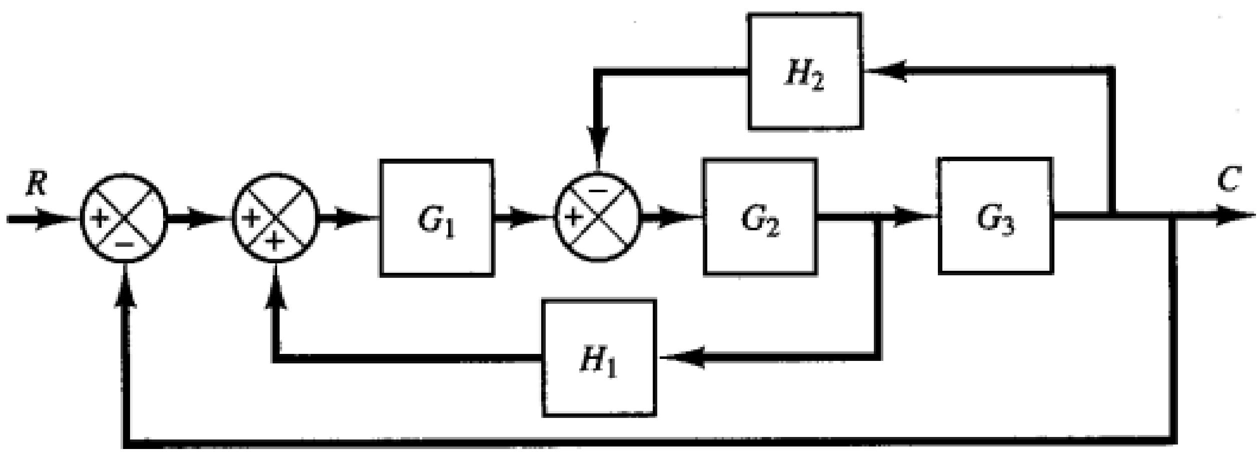
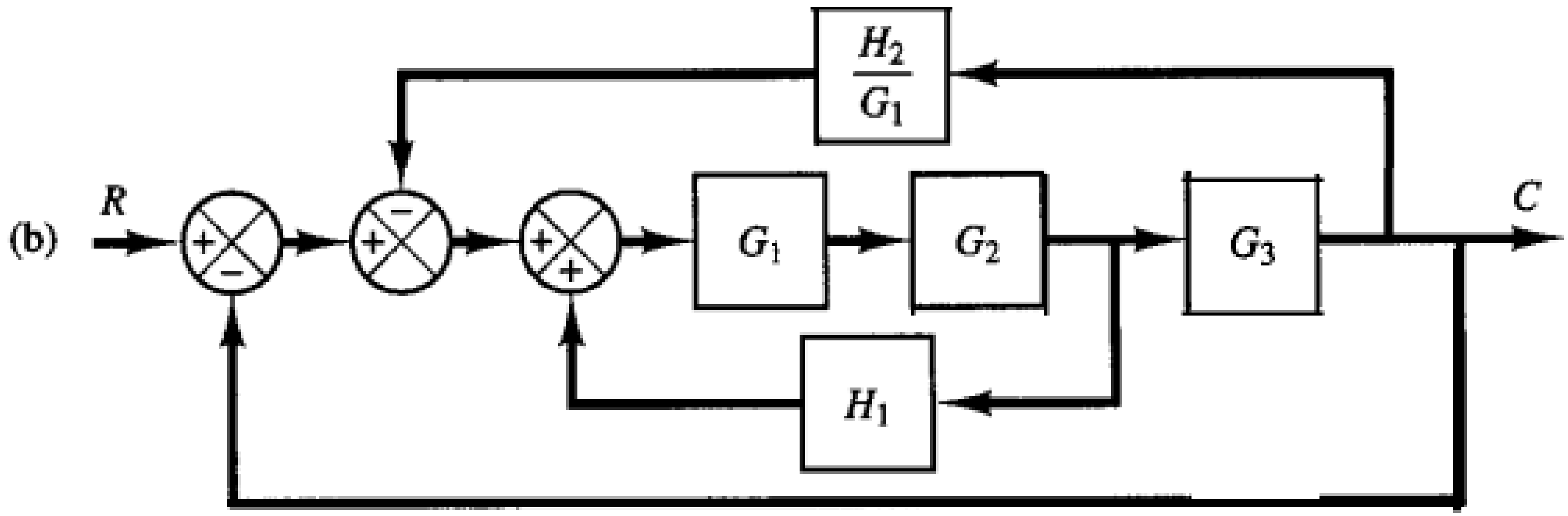


Diagrama de Bloques



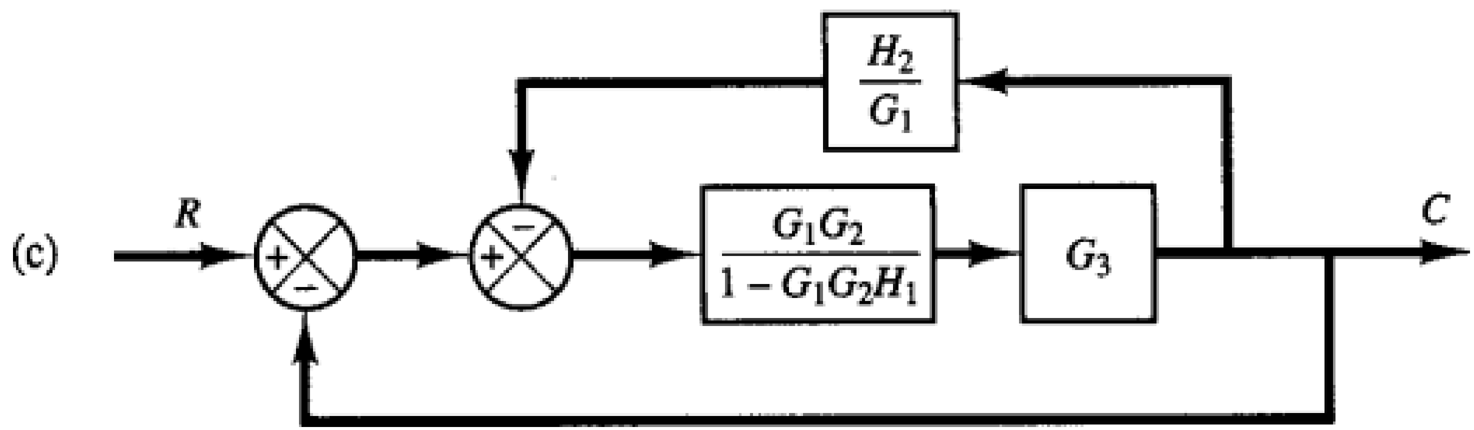
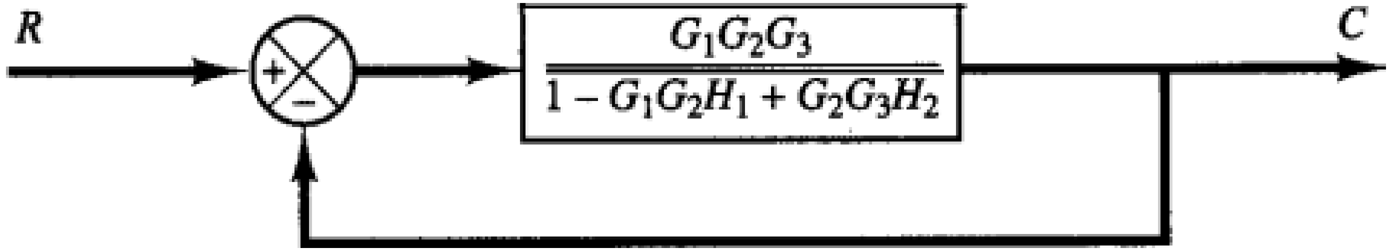
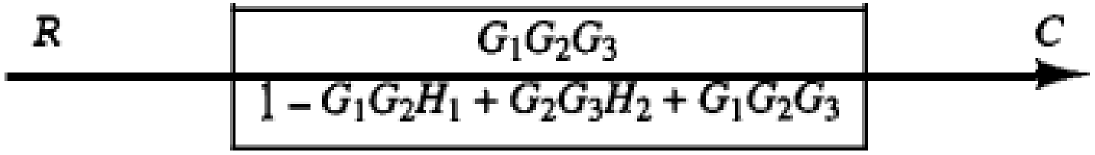


Diagrama de Bloques

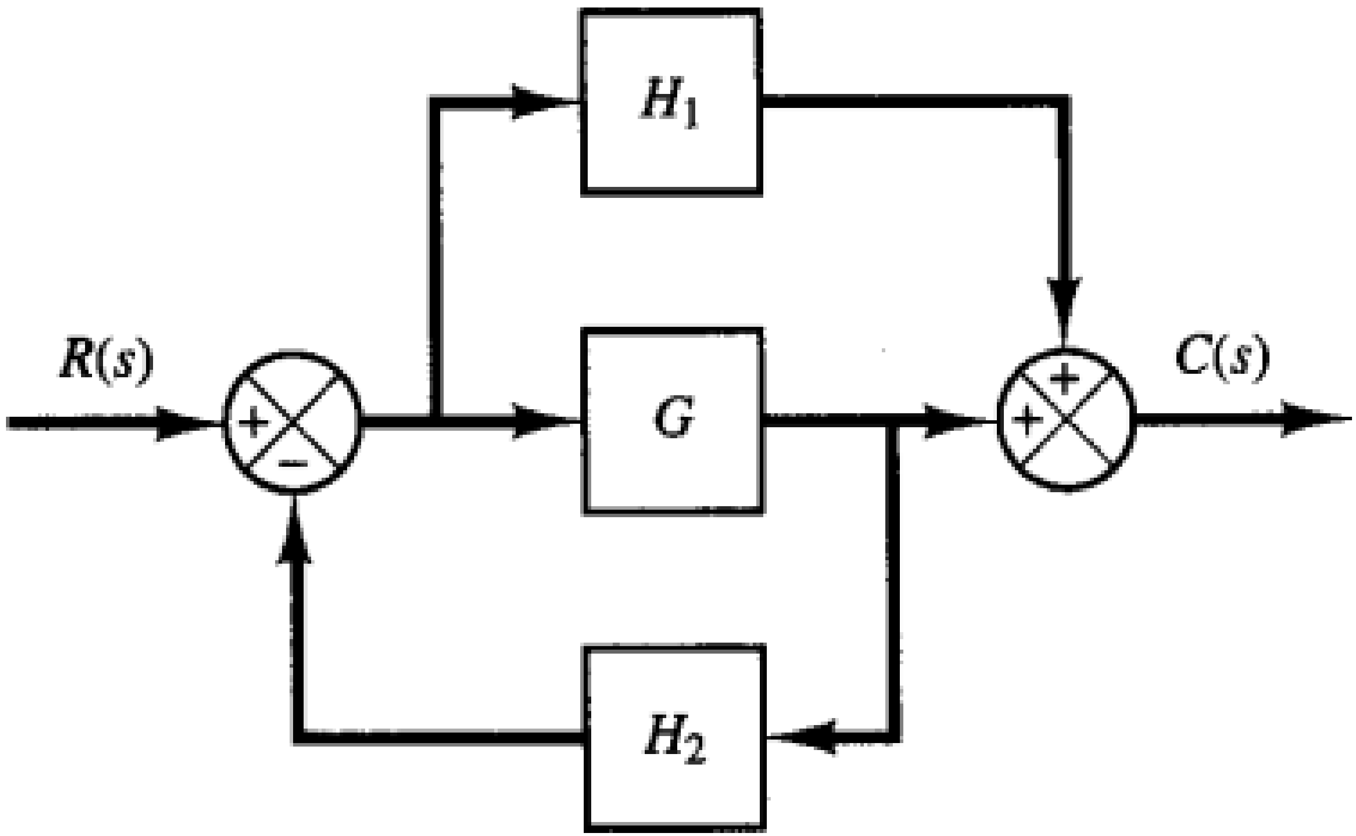
(d)



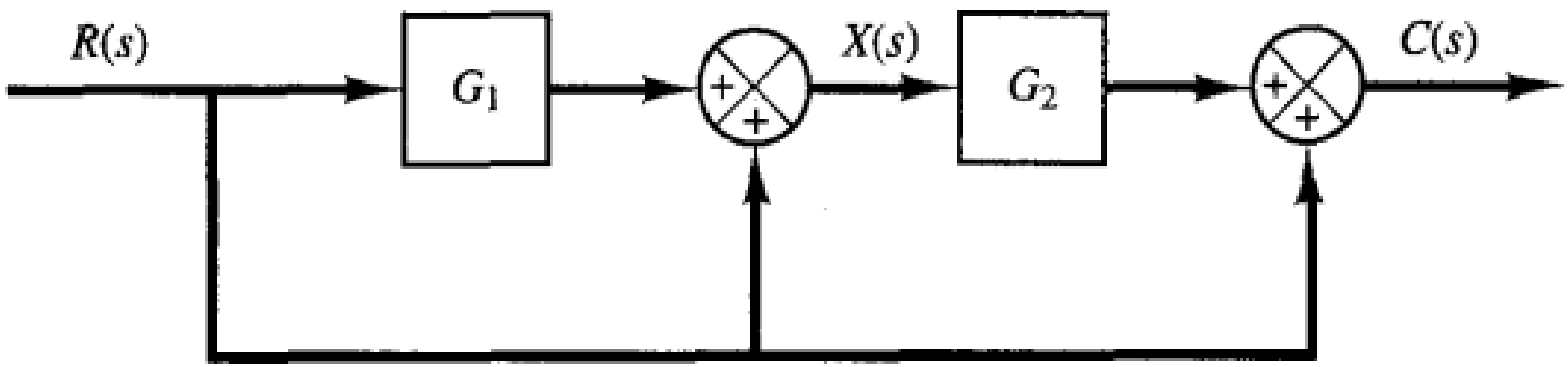
(e)



Ejercicios



Ejercicios



Ejercicios

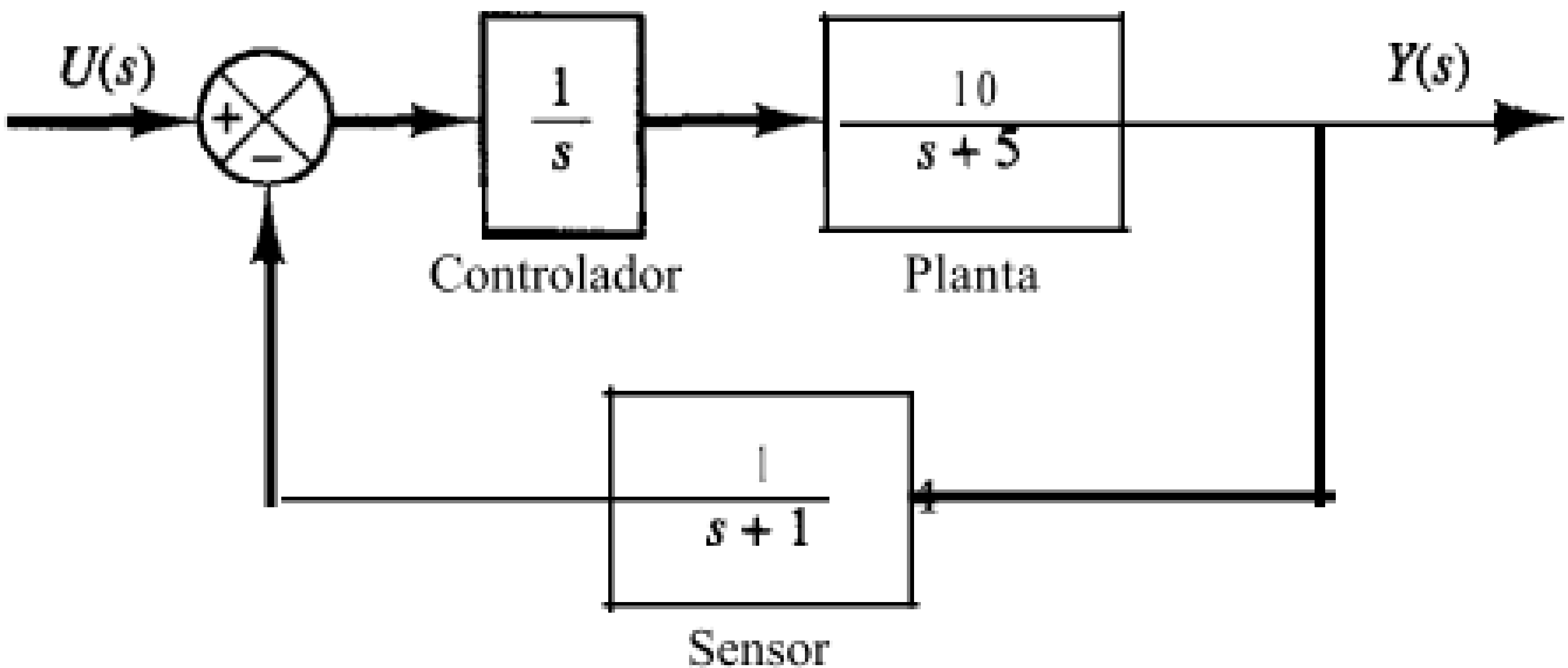


Diagrama de Flujo de Señales

Consider a linear system that is described by a set of N algebraic equations:

$$y_j = \sum_{k=1}^{N} a_{kj} y_k \quad j = 1, 2, \ldots, N \tag{3-44}$$

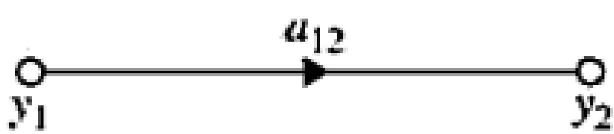


Figure 3-24 Signal flow graph of $y_2 = a_{12} y_1$.

Diagrama de Flujo de Señales

As an example on the construction of an SFG, consider the following set of algebraic equations:

$$y_2 = a_{12}y_1 + a_{32}y_3$$
$$y_3 = a_{23}y_2 + a_{43}y_4$$
$$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4 \quad (3\text{-}50)$$
$$y_5 = a_{25}y_2 + a_{45}y_4$$

The SFG for these equations is constructed, step by step, in Fig. 3-25.

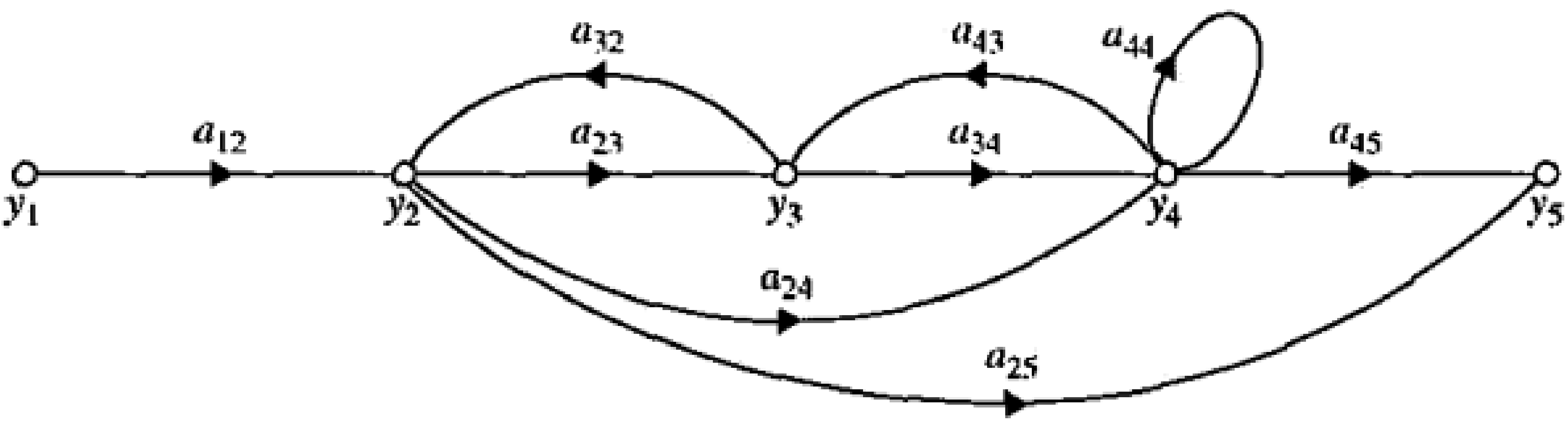


Diagrama de Flujo de Señales

Input Node (Source): *An* input node *is a node that has only outgoing branches* (example: node y_1 in Fig. 3-24).

Output Node (Sink): *An* output node *is a node that has only incoming branches:* (example: node y_2 in Fig. 3-24). However, this condition is not always readily met by an output node. For instance, the SFG in Fig. 3-26(a) does not have a node that satisfies the

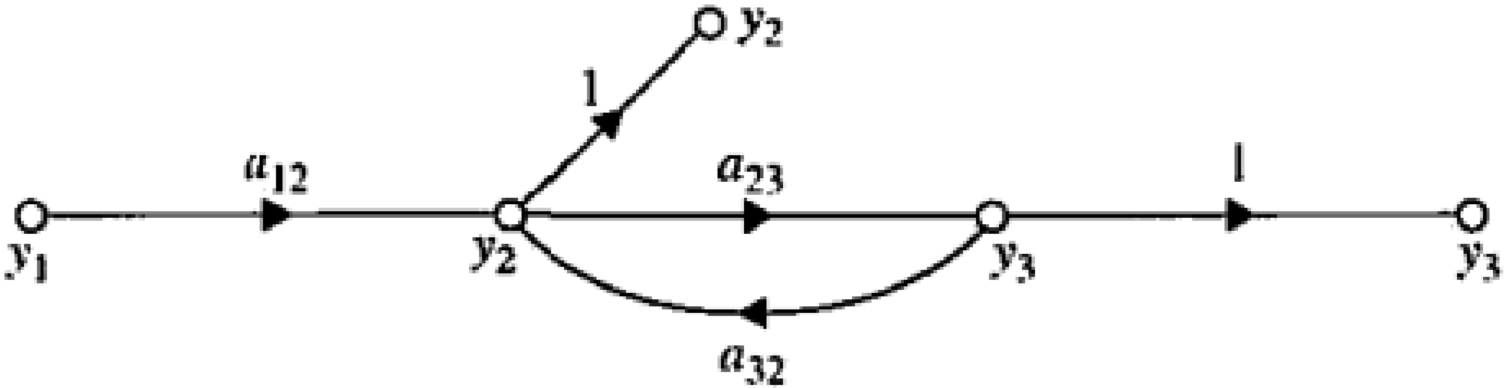


Diagrama de Flujo de Señales

Path: *A* path *is any collection of a continuous succession of branches traversed in the same direction.* The definition of a path is entirely general, since it does not prevent any node from being traversed more than once. Therefore, as simple as the SFG of Fig. 3-26(a) is, it may have numerous paths just by traversing the branches a_{23} and a_{32} continuously.

Forward Path: *A* forward path *is a path that starts at an input node and ends at an output node and along which no node is traversed more than once.* For example, in the SFG of Fig. 3-25(d), y_1 is the input node, and the rest of the nodes are all possible output

Diagrama de Flujo de Señales

Path Gain: *The product of the branch gains encountered in traversing a path is called the* path gain. For example, the path gain for the path $y_1 - y_2 - y_3 - y_4$ in Fig. 3-25(d) is $a_{12}a_{23}a_{34}$.

Loop: *A* loop *is a path that originates and terminates on the same node and along which no other node is encountered more than once*. For example, there are four loops in the SFG of Fig. 3-25(d). These are shown in Fig. 3-28.

Forward-Path Gain: *The* forward-path gain *is the path gain of a forward path.*

Diagrama de Flujo de Señales

Loop Gain: *The* loop gain *is the path gain of a loop*. For example, the loop gain of the loop $y_2 - y_4 - y_3 - y_2$ in Fig. 3-28 is $a_{24}a_{43}a_{32}$.

Nontouching Loops: *Two parts of an SFG are* nontouching *if they do not share a common node*. For example, the loops $y_2 - y_3 - y_2$ and $y_4 - y_4$ of the SFG in Fig. 3-25(d) are nontouching loops.

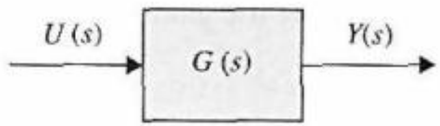
Diagrama de Flujo de Señales

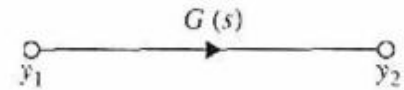
TABLE 3-1 Block diagrams and their SFG equivalent representations

Block Diagram | Signal Flow Diagram

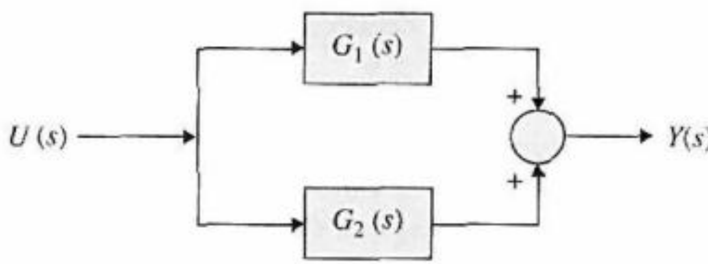
Simple Transfer Function

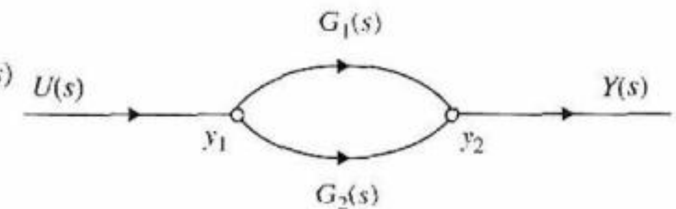
$$\frac{Y(s)}{U(s)} = G(s)$$



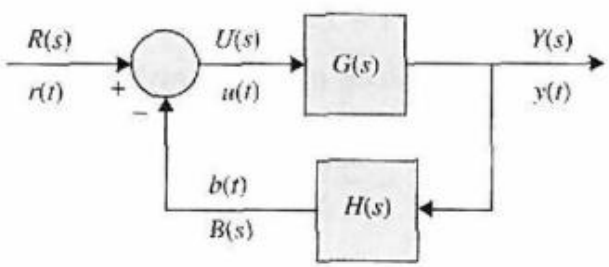


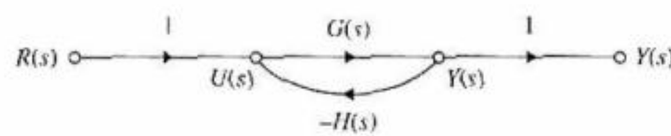
Parallel Feedback





$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$





Teorema de Mason

Samuel Jefferson Mason (1921–1974) was an American electronics engineer. Mason's invariant andMason's rule are named after him.

He was born in New York City, but he grew up in a small town[which?] in New Jersey. It was so small, in fact, that it only had a population of 26. He received a B.S. in electrical engineering from Rutgers University in 1942, and he joined the Antenna Group of MIT's Radiation Laboratory as a staff member after graduation. Mason went on to earn his S.M. and Ph.D. in electrical engineering from MIT in 1947 and 1952, respectively.[1] After World War II, MIT's Radiation Laboratory was renamed the MIT Research Laboratory of Electronics, and he became the associate director of the laboratory in 1967.[2] Mason served on the faculty of MIT from 1949 until his death in 1974 – as an assistant professor in 1949, associate professor in 1954, and full professor in 1959.[1] Mason unexpectedly died in 1974 due to a cerebral hemorrhage.[3]

Mason's doctoral dissertation, supervised by Ernst Guillemin[4] , was on signal-flow graphs and he is often credited with inventing them[5].

Teorema de Mason

Given an SFG with N forward paths and K loops, the gain between the input node y_{in} and output node y_{out} is [3]

$$M = \frac{y_{\text{out}}}{y_{\text{in}}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta} \tag{3-54}$$

where

y_{in} = input-node variable

y_{out} = output-node variable

M = gain between y_{in} and y_{out}

N = total number of forward paths between y_{in} and y_{out}

M_k = gain of the kth forward paths between y_{in} and y_{out}

$$\Delta = 1 - \sum_i L_{i1} + \sum_j L_{j2} - \sum_k L_{k3} + \dots \tag{3-55}$$

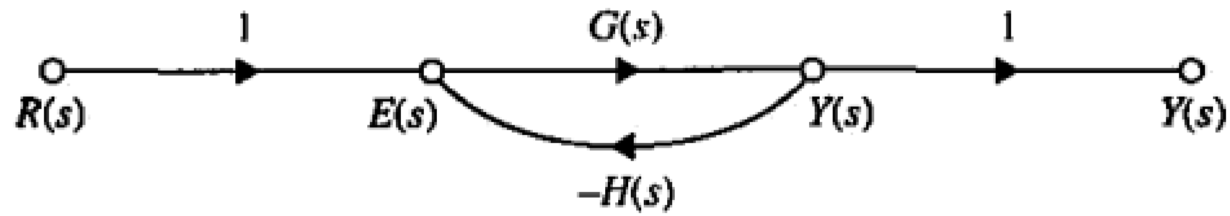
L_{mr} = gain product of the mth $(m = i, j, k, \dots)$ possible combination of r nontouching loops $(1 \le r \le K)$.

or

Δ = 1 − (sum of the gains of **all individual** loops) + (sum of products of gains of all possible combinations of **two** nontouching loops) − (sum of products of gains of all possible combinations of **three** nontouching loops) + ⋯

Δ_k = the Δ for that part of the SFG that is nontouching with the kth forward path.

Teorema de Mason



1. There is only one forward path between $R(s)$ and $Y(s)$, and the forward-path gain is

$$M_1 = G(s) \tag{3-56}$$

2. There is only one loop; the loop gain is

$$L_{11} = -G(s)H(s) \tag{3-57}$$

3. There are no nontouching loops since there is only one loop. Furthermore, the forward path is in touch with the only loop. Thus, $\Delta_1 = 1$, and

$$\Delta = 1 - L_{11} = 1 + G(s)H(s) \tag{3-58}$$

Using Eq. (3-54), the closed-loop transfer function is written

$$\frac{Y(s)}{R(s)} = \frac{M_1\Delta_1}{\Delta} = \frac{G(s)}{1 + G(s)H(s)} \tag{3-59}$$

which agrees with Eq. (3-12).

Teorema de Mason

EXAMPLE 3-2-3 Consider the SFG shown in Fig. 3-25(d). Let us first determine the gain between y_1 and y_5 using the gain formula.

The three forward paths between y_1 and y_5 and the forward-path gains are

$$M_1 = a_{12}a_{23}a_{34}a_{45} \quad \text{Forward path:} \quad y_1 - y_2 - y_3 - y_4 - y_5$$
$$M_2 = a_{12}a_{25} \quad \text{Forward path:} \quad y_1 - y_2 - y_5$$
$$M_3 = a_{12}a_{24}a_{45} \quad \text{Forward path:} \quad y_1 - y_2 - y_4 - y_5$$

The four loops of the SFG are shown in Fig. 3-28. The loop gains are

$$L_{11} = a_{23}a_{32} \quad L_{21} = a_{34}a_{43} \quad L_{31} = a_{24}a_{43}a_{32} \quad L_{41} = a_{44}$$

There is only one pair of nontouching loops; that is, the two loops are

$$y_2 - y_3 - y_2 \quad \text{and} \quad y_4 - y_4$$

Thus, the product of the gains of the two nontouching loops is

$$L_{12} = a_{23}a_{32}a_{44} \tag{3-60}$$

All the loops are in touch with forward paths M_1 and M_3. Thus, $\Delta_1 = \Delta_3 = 1$. Two of the loops are not in touch with forward path M_2. These loops are $y_3 - y_4 - y_3$ and $y_4 - y_4$. Thus,

$$\Delta_2 = 1 - a_{34}a_{43} - a_{44} \tag{3-61}$$

Substituting these quantities into Eq. (3-54), we have

$$\begin{aligned}\frac{y_5}{y_1} &= \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta} \\ &= \frac{(a_{12}a_{23}a_{34}a_{45}) + (a_{12}a_{25})(1 - a_{34}a_{43} - a_{44}) + a_{12}a_{24}a_{45}}{1 - (a_{23}a_{32} + a_{34}a_{43} + a_{24}a_{32}a_{43} + a_{44}) + a_{23}a_{32}a_{44}}\end{aligned} \tag{3-62}$$

EXAMPLE 3-2-4 Consider the SFG in Fig. 3-33. The following input–output relations are obtained by use of the gain formula:

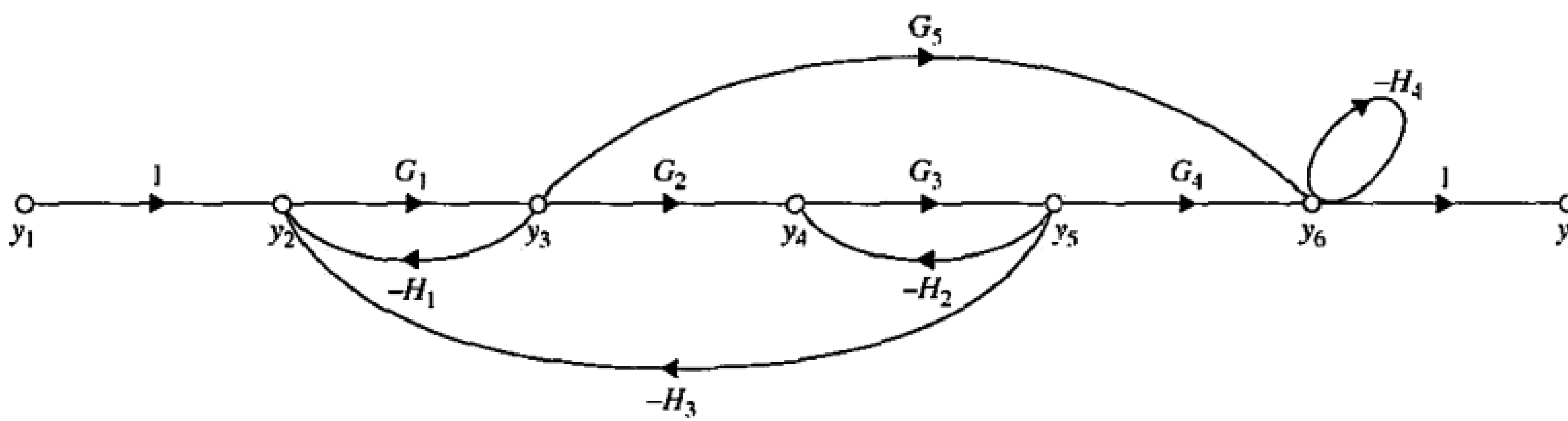
$$\frac{y_2}{y_1} = \frac{1 + G_3H_2 + H_4 + G_3H_2H_4}{\Delta} \tag{3-65}$$

$$\frac{y_4}{y_1} = \frac{G_1G_2(1 + H_4)}{\Delta} \tag{3-66}$$

$$\frac{y_6}{y_1} = \frac{y_7}{y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1 + G_3H_2)}{\Delta} \tag{3-67}$$

where

$$\begin{aligned}\Delta = 1 &+ G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_1G_3H_1H_2 \\ &+ G_1H_1H_4 + G_3H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_3H_1H_2H_4\end{aligned} \tag{3-68}$$



3-3. Reduce the block diagram shown in Fig. 3P-3 and find the Y/X.

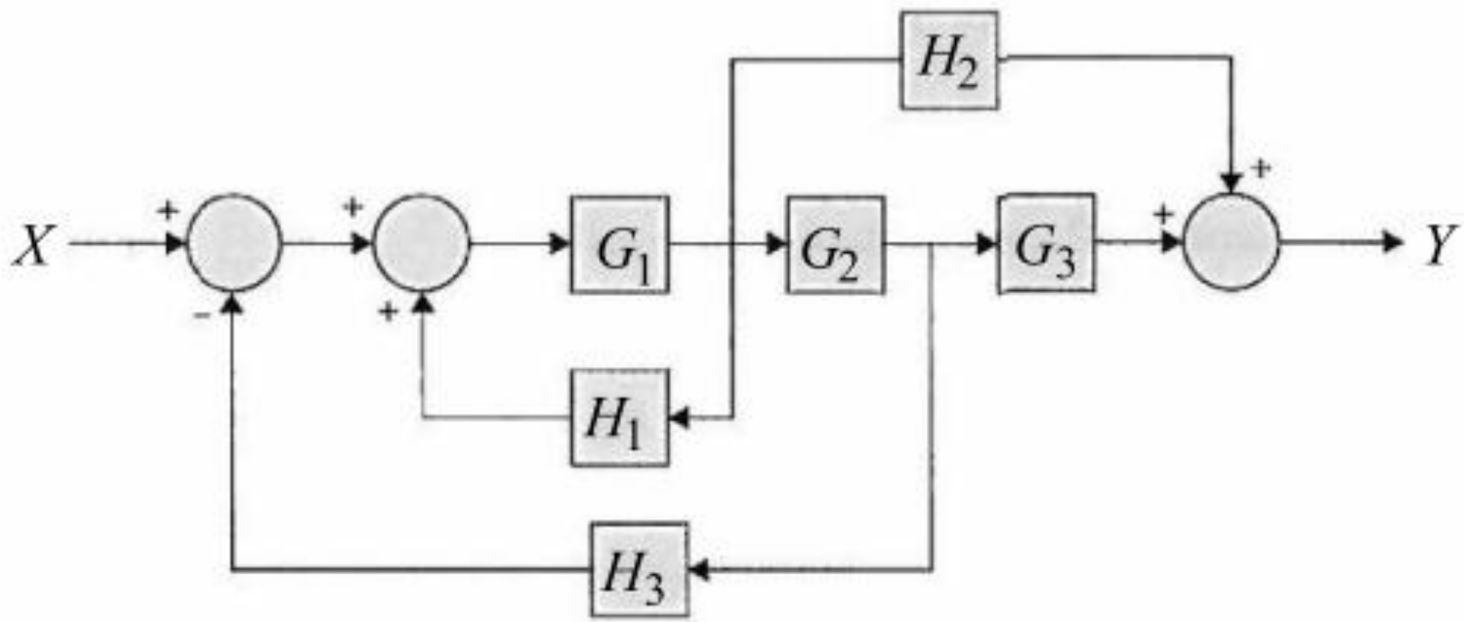
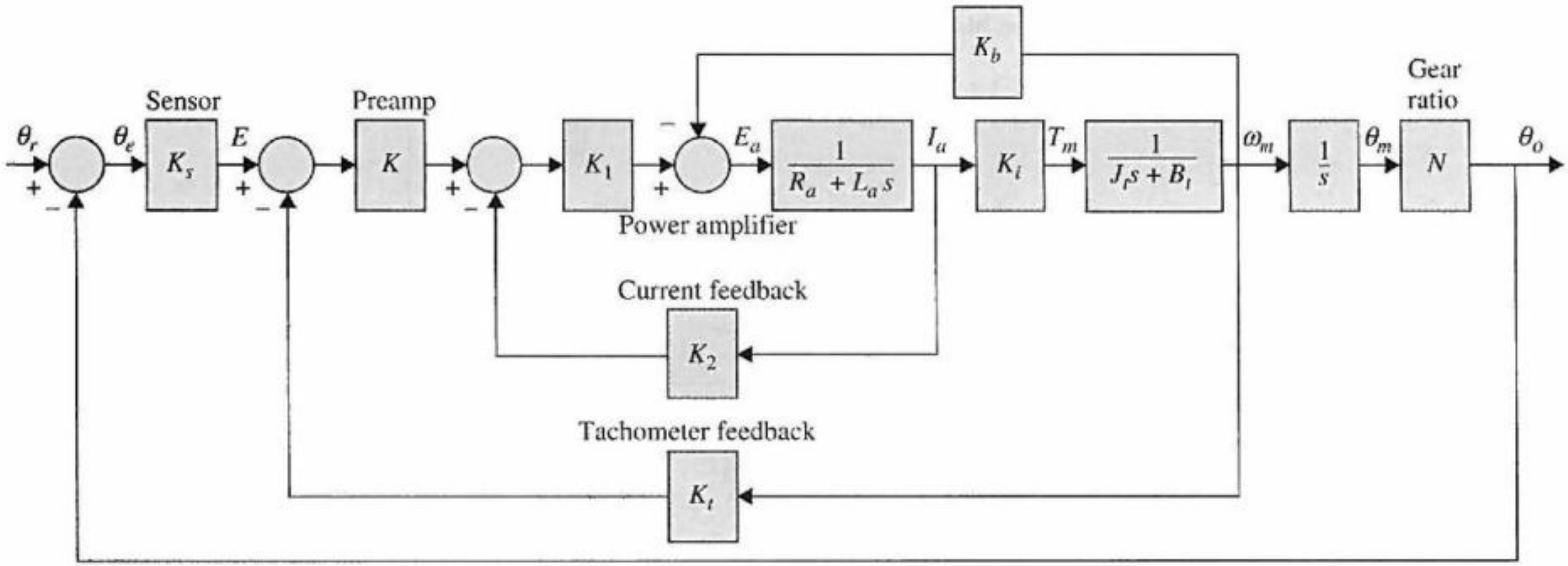
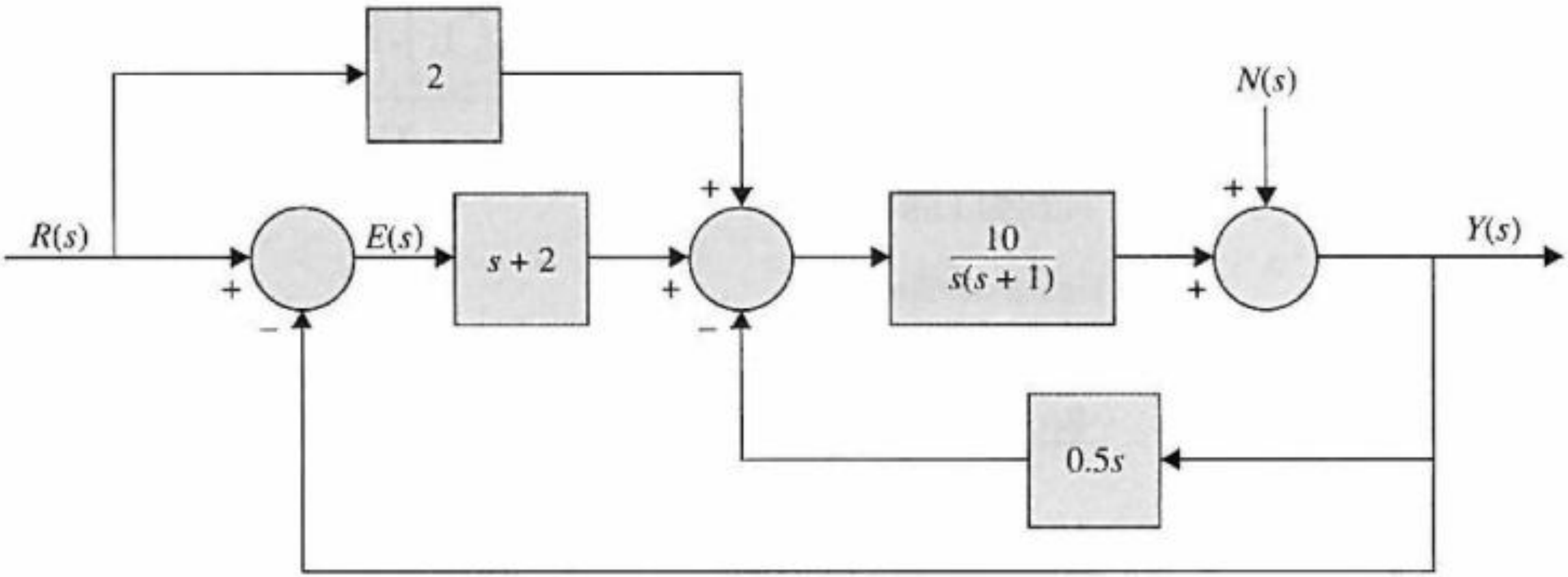


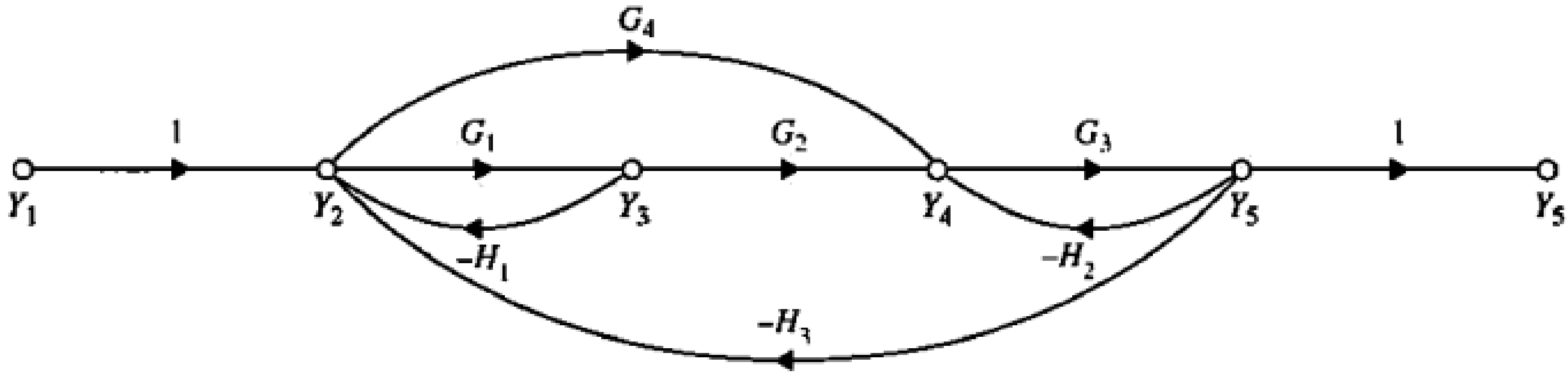
Figure 3P-3



K_b
Gear ratio
Sensor
Preamp
θ_r
θ_e
K_s
E
K
K_1
E_a
$\frac{1}{R_a + L_a s}$
I_a
K_i
T_m
$\frac{1}{J_t s + B_t}$
ω_m
$\frac{1}{s}$
θ_m
N
θ_o
Power amplifier
Current feedback
K_2
Tachometer feedback
K_t

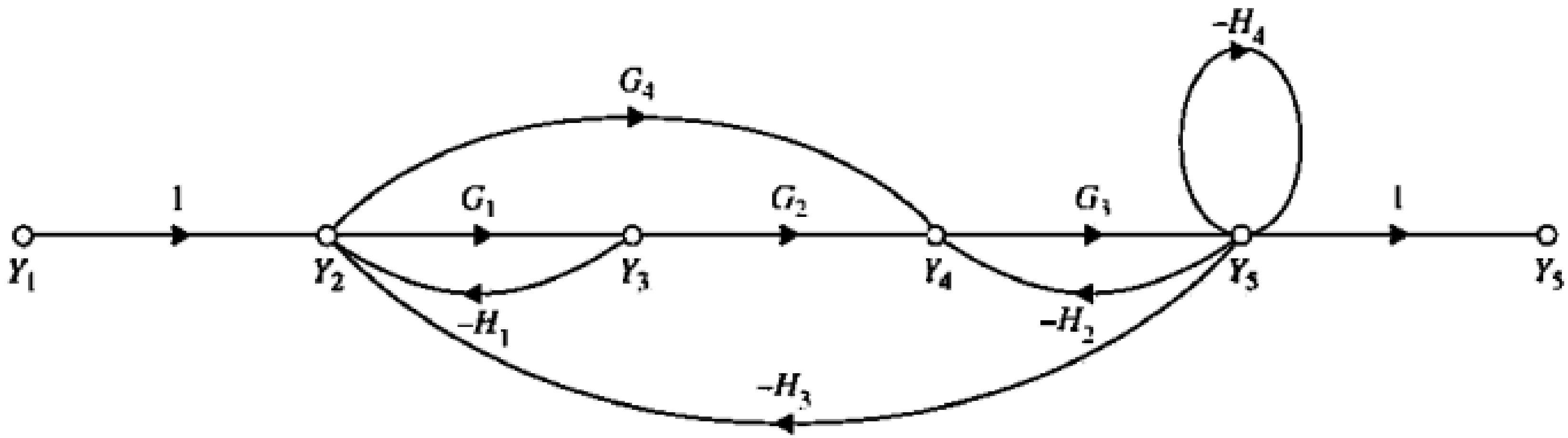
Figure 3P-7


2
N(s)
R(s)
E(s)
s + 2
10/s(s + 1)
Y(s)
0.5s
+
−
+
+
−
+
+



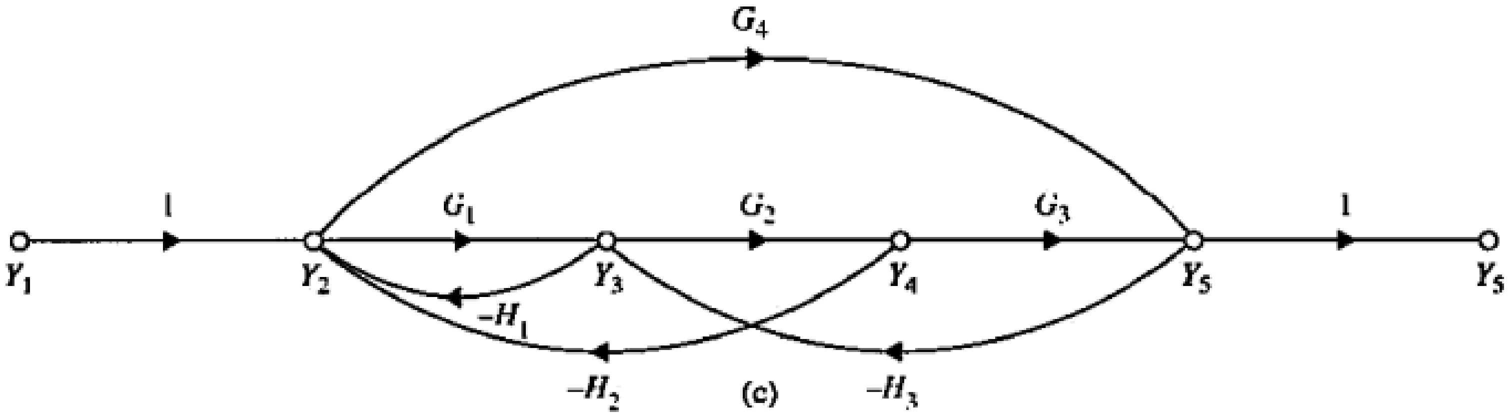
G_4
1
G_1
G_2
G_3
1
Y_1
Y_2
Y_3
Y_4
Y_5
Y_5
$-H_1$
$-H_2$
$-H_3$

(a)



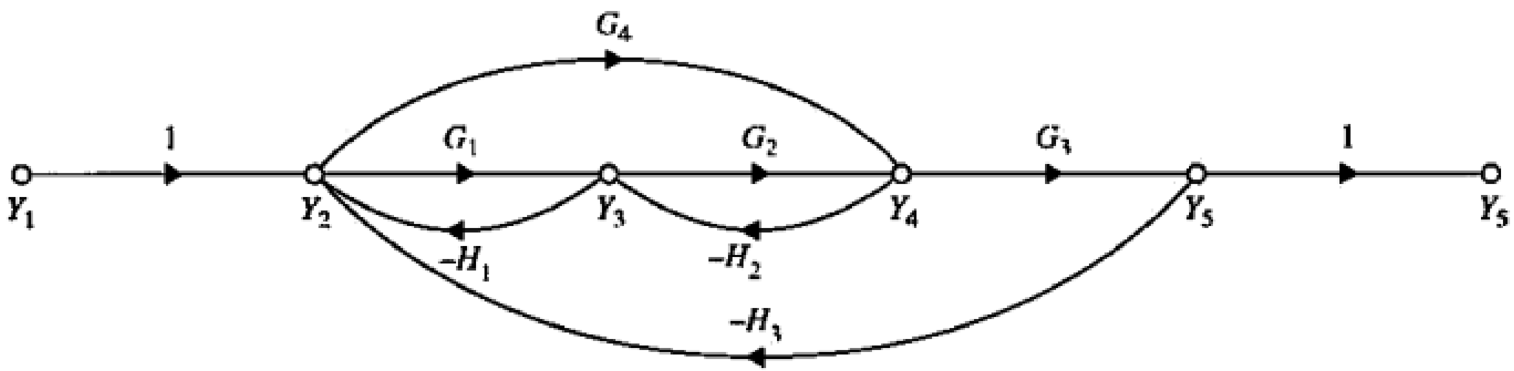
$-H_4$
G_4
1
G_1
G_2
G_3
1
Y_1
Y_2
Y_3
Y_4
Y_5
Y_5
$-H_1$
$-H_2$
$-H_3$

(b)



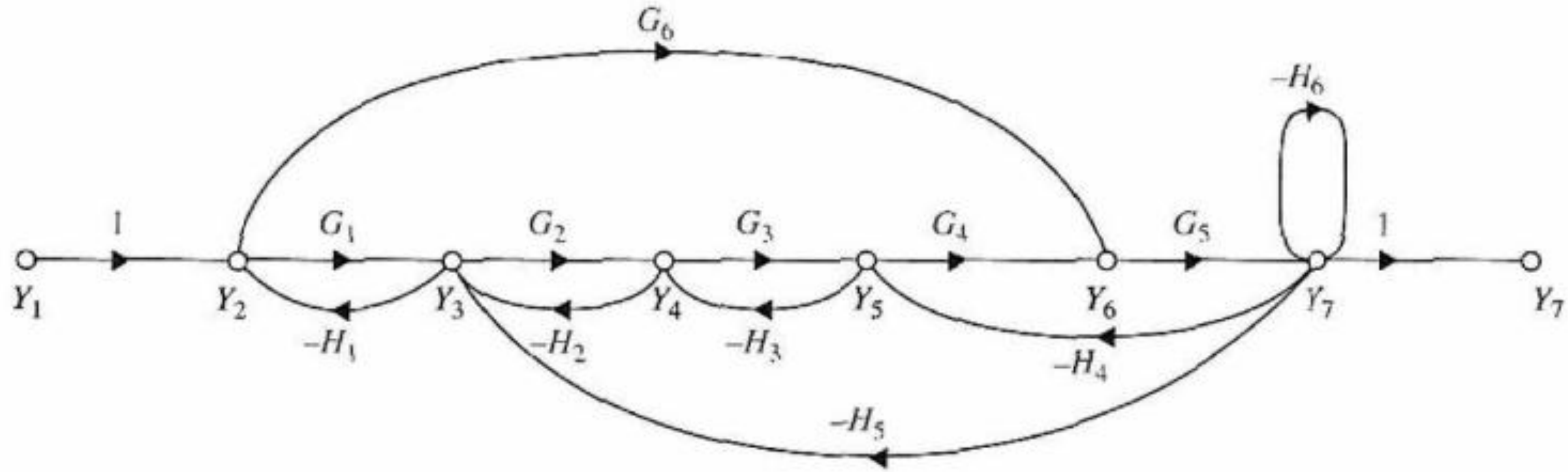
G_4
1
G_1
G_2
G_3
1
Y_1
Y_2
Y_3
Y_4
Y_5
Y_5
$-H_1$
$-H_2$
$-H_3$

(c)

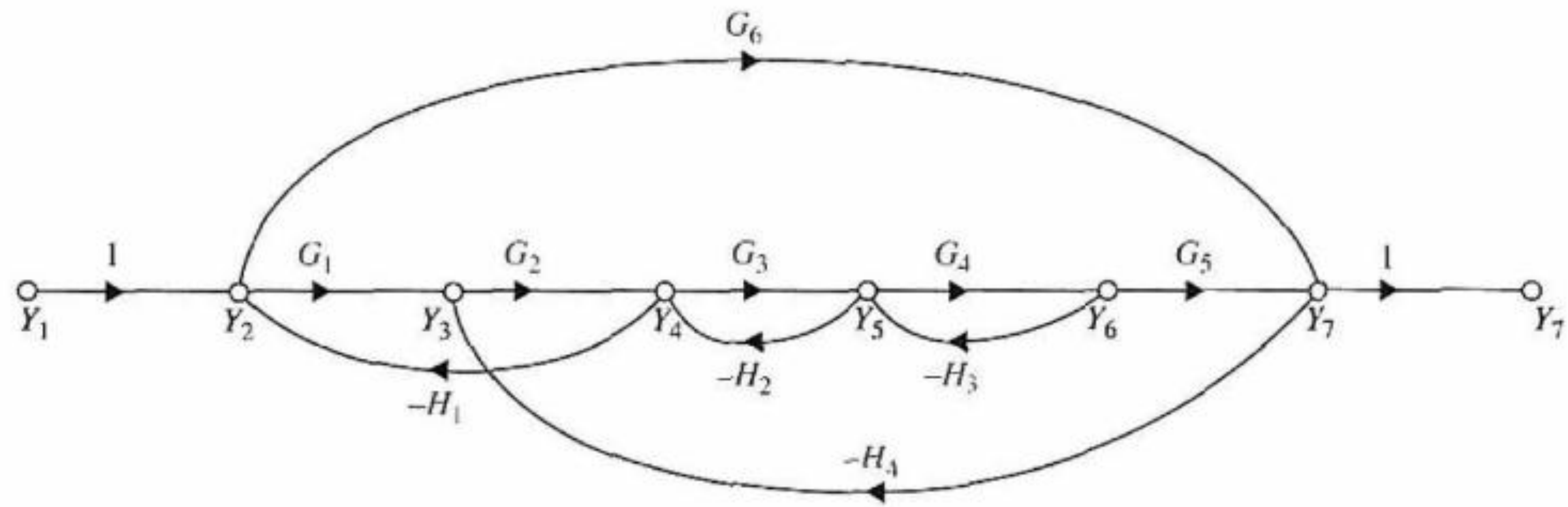


G_4
1
G_1
G_2
G_3
1
Y_1
Y_2
Y_3
Y_4
Y_5
Y_5
$-H_1$
$-H_2$
$-H_3$

(d)


G6
-H6
1
G1
G2
G3
G4
G5
1
Y1
Y2
Y3
Y4
Y5
Y6
Y7
Y7
-H1
-H2
-H3
-H4
-H5

(a)


G6
1
G1
G2
G3
G4
G5
1
Y1
Y2
Y3
Y4
Y5
Y6
Y7
Y7
-H2
-H3
-H1
-H4